Multilevel Workshop

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1 Introduction

This site contains materials for a workshop on multilevel modeling.

Multilevel models are useful when you have data that are nested or clustered inside social units such as schools, neighborhoods, states, or countries.

Multilevel models are also useful when you have longitudinal data where repeated measures are collected for study participants.

2 Two Level Cross Sectional; And Three Level Longitudinal Models

2.1 Cross Sectional Model

2.1.1 Get Data

```
use "simulated_multilevel_data.dta", clear
```

2.1.2 The Equation

$$\text{outcome}_{ij} = \beta_0 + \beta_1 \text{parental warmth} + \beta_2 \text{physical punishment} + \beta_3 \text{time} +$$

$$\beta_4 \text{identity}_2 + \beta_5 \text{intervention} + \beta_6 HDI +$$

$$u_{0j} + u_{1j} \times \text{parental warmth} + e_{ij}$$

2.1.3 Descriptive Statistics

```
summarize // descriptive statistics
```

```
    Variable |        Obs        Mean    Std. dev.       Min        Max
-------------+---------------------------------------------------------
     country |      3,000        15.5    8.656884          1         30
         HDI |      3,000    64.76667    17.24562         33         87
      family |      3,000        50.5    28.87088          1        100
          id |          0
    identity |      3,000    .4976667    .5000779          0          1
```

```
-------------+---------------------------------------------------------
intervention |      3,000    .4843333    .4998378          0          1
physical_p~t |      3,000    2.478667    1.360942          0          5
      warmth |      3,000    3.521667    1.888399          0          7
     outcome |      3,000    52.43327    6.530996   29.60798   74.83553
```

2.1.4 Spaghetti Plot

```
spagplot outcome warmth, id(country) scheme(stcolor)

graph export spagplot1.png, width(1000) replace
```

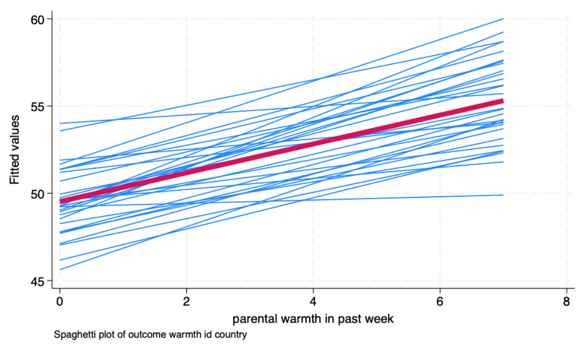

Figure 2.1: Spaghetti Plot of Outcome by Warmth by Country

2.1.5 Unconditional Model

2.1.5.1 Model

```
mixed outcome || country: // unconditional model
```

```
Performing EM optimization ...

Performing gradient-based optimization:
Iteration 0:  Log likelihood = -9802.8371
Iteration 1:  Log likelihood = -9802.8371

Computing standard errors ...
```

```
Mixed-effects ML regression                          Number of obs    = 3,000
Group variable: country                              Number of groups =    30
                                                     Obs per group:
                                                                  min =   100
                                                                  avg = 100.0
                                                                  max =   100
                                                     Wald chi2(0)     =     .
Log likelihood = -9802.8371                          Prob > chi2      =     .

------------------------------------------------------------------------------
     outcome | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
       _cons |   52.43327   .3451217   151.93   0.000     51.75685     53.1097
------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
country: Identity            |
                  var(_cons) |   3.178658   .9226737      1.799552    5.614658
-----------------------------+------------------------------------------------
               var(Residual) |   39.46106   1.024013      37.50421       41.52
------------------------------------------------------------------------------
LR test vs. linear model: chibar2(01) = 166.31        Prob >= chibar2 = 0.0000
```

2.1.5.2 ICC

```
estat icc
```

```
Intraclass correlation

------------------------------------------------------------------------------
                       Level |        ICC   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
                     country |   .0745469   .0201254      .0434963    .1248696
------------------------------------------------------------------------------
```

2.1.6 Conditional Model

```
mixed outcome warmth physical_punishment identity i.intervention HDI || country: warmth // mu

est store crosssectional // store estimates
```

```
Performing EM optimization ...

Performing gradient-based optimization:
Iteration 0:  Log likelihood = -9626.6279
Iteration 1:  Log likelihood =  -9626.607
Iteration 2:  Log likelihood =  -9626.607

Computing standard errors ...

Mixed-effects ML regression                     Number of obs    =  3,000
Group variable: country                         Number of groups =     30
                                                Obs per group:
                                                             min =    100
                                                             avg =  100.0
                                                             max =    100
                                                Wald chi2(5)     = 334.14
Log likelihood =  -9626.607                     Prob > chi2      = 0.0000

------------------------------------------------------------------------------------
            outcome | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
--------------------+---------------------------------------------------------------
             warmth |   .8345368   .0637213    13.10   0.000     .7096453    .9594282
physical_punishment |  -.9916657   .0797906   -12.43   0.000    -1.148052   -.8352791
           identity |  -.3004767   .2170295    -1.38   0.166    -.7258466    .1248933
     1.intervention |   .6396427   .2174519     2.94   0.003     .2134448    1.065841
                HDI |   -.003228   .0199257    -0.16   0.871    -.0422817    .0358256
              _cons |   51.99991   1.371257    37.92   0.000      49.3123    54.68753
------------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
country: Independent         |
                var(warmth)  |   .0227504   .0257784      .0024689    .2096436
                 var(_cons)  |   2.963975   .9737647      1.556777    5.643163
```

```
-----------------------------+------------------------------------------------
               var(Residual) |   34.97499   .9097109      33.23668    36.80422
------------------------------------------------------------------------------
LR test vs. linear model: chi2(2) = 205.74                Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

2.2 Longitudinal Model

2.2.1 Get Data

```
use "simulated_multilevel_longitudinal_data.dta", clear
```

2.2.2 The Equation

$$\text{outcome}_{ij} = \beta_0 + \beta_1 \text{parental warmth} + \beta_2 \text{physical punishment} + \beta_3 \text{time} +$$

$$\beta_4 \text{identity}_2 + \beta_5 \text{intervention} + \beta_5 HDI +$$

$$u_{0j} + u_{1j} \times \text{parental warmth} +$$

$$v_{0i} + v_{1i} \times t + e_{ij}$$

2.2.3 Descriptive Statistics

```
summarize // descriptive statistics
```

```
    Variable |        Obs        Mean    Std. dev.       Min        Max
-------------+---------------------------------------------------------
     country |      9,000        15.5    8.655922          1         30
         HDI |      9,000    64.76667     17.2437         33         87
      family |      9,000        50.5    28.86767          1        100
          id |          0
```

```
    identity |      9,000    .4976667    .5000223          0          1
-------------+---------------------------------------------------------
intervention |      9,000    .4843333    .4997823          0          1
           t |      9,000           2    .8165419          1          3
physical_p~t |      9,000    2.485333    1.373639          0          5
      warmth |      9,000    3.514222      1.8839          0          7
     outcome |      9,000    53.37768    6.572285   29.60798   79.02199
```

2.2.4 Alternate Plot

```
encode id, generate(idNUMERIC) // numeric version of id

* spagplot outcome t if idNUMERIC <= 10, id(idNUMERIC) scheme(stcolor)

twoway (lfit outcome t) (scatter outcome t) if idNUMERIC <= 10, by(idNUMERIC) scheme(stcolor)

graph export spagplot2.png, width(1000) replace
```

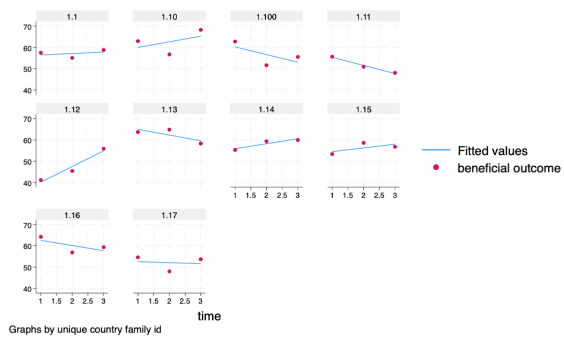

Figure 2.2: Alternate Plot of Outcome by Time by Individual; First 10 Observations

2.2.5 Unconditional Model

2.2.5.1 Model

```
mixed outcome || country: || id: // unconditional model
```

2.2.5.2 ICC

```
estat icc
```

```
Intraclass correlation

------------------------------------------------------------------------------
                       Level |        ICC   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
                     country |   .0748336   .0190847      .0450028    .1219141
                  id|country |   .3462837   .0171461      .3134867    .3806097
------------------------------------------------------------------------------
```

2.2.6 Conditional Model

```
mixed outcome t warmth physical_punishment i.identity i.intervention HDI || country: warmth

est store longitudinal // store estimates
```

```
Performing EM optimization ...

Performing gradient-based optimization:
Iteration 0:  Log likelihood =  -28523.49
Iteration 1:  Log likelihood = -28499.987
Iteration 2:  Log likelihood = -28499.739
Iteration 3:  Log likelihood = -28499.604
Iteration 4:  Log likelihood = -28499.603

Computing standard errors ...

Mixed-effects ML regression                          Number of obs =   9,000

        Grouping information
        -------------------------------------------------------------
                        |     No. of       Observations per group
         Group variable |     groups    Minimum    Average    Maximum
        ----------------+--------------------------------------------
                country |         30        300      300.0        300
```

```
                 id |      3,000          3        3.0          3
        -------------------------------------------------------------

                                                        Wald chi2(6)  = 1096.15
Log likelihood = -28499.603                             Prob > chi2   =  0.0000

-------------------------------------------------------------------------------------
            outcome | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
--------------------+----------------------------------------------------------------
                  t |    .943864   .0658716    14.33   0.000      .814758     1.07297
             warmth |   .9134959   .0423732    21.56   0.000      .830446    .9965459
physical_punishment |  -1.007897   .0497622   -20.25   0.000    -1.105429   -.9103647
         1.identity |  -.1276926   .1515835    -0.84   0.400    -.4247908    .1694057
     1.intervention |   .8589966   .1519095     5.65   0.000     .5612596    1.156734
                HDI |  -.0005657   .0196437    -0.03   0.977    -.0390666    .0379352
              _cons |   50.46724   1.338318    37.71   0.000     47.84418    53.09029
-------------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
country: Independent         |
                 var(warmth) |   .0107586   .0127845      .0010478    .1104703
                  var(_cons) |   3.167085   .9146761      1.798154    5.578181
-----------------------------+------------------------------------------------
id: Independent              |
                      var(t) |   3.58e-09   7.06e-07      3.5e-177    3.7e+159
                  var(_cons) |   8.387275   .4724188      7.510631    9.366242
-----------------------------+------------------------------------------------
               var(Residual) |   26.02733   .4753701      25.11211    26.97592
------------------------------------------------------------------------------
LR test vs. linear model: chi2(4) = 1247.03               Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

2.3 Nice Table of Results

```
etable, estimates(crosssectional longitudinal) ///
showstars showstarsnote /// show stars and note
column(estimate) // column is modelname
```

	crosssectional	longitudinal
parental warmth in past week	0.835 **	0.913 **
	(0.064)	(0.042)
physical punishment in past week	-0.992 **	-1.008 **
	(0.080)	(0.050)
hypothetical identity group variable	-0.300	
	(0.217)	
recieved intervention		
1	0.640 **	0.859 **
	(0.217)	(0.152)
Human Development Index	-0.003	-0.001
	(0.020)	(0.020)
time		0.944 **
		(0.066)
hypothetical identity group variable		
1		-0.128
		(0.152)
Intercept	52.000 **	50.467 **
	(1.371)	(1.338)
var(warmth)	0.023	0.011
	(0.026)	(0.013)
var(_cons)	2.964	3.167
	(0.974)	(0.915)
var(e)	34.975	26.027
	(0.910)	(0.475)
var(_cons)		8.387
		(0.472)
var(t)		0.000
		(0.000)
Number of observations	3000	9000

** p<.01, * p<.05

2.4 QUESTIONS???

3 Cross-Classified Models

3.1 Introduction

A two level multilevel model imagines that *Level 1* units are nested in *Level 2* units. A three level multilevel model imagines that *Level 1* units are nested in *Level 2* units, which are in turn nested in *Level 3*.

A cross-classified model imagines that the nesting is not hierarchical, but rather that there are two sets of clusters or nestings in which individuals may be nested.

3.2 Get Data

```
use "simulated_multilevel_longitudinal_data.dta", clear
```

3.3 Cross Classified Model

We can treat these random effects as being *cross classified*.

This might be useful if we had data where individuals lived in different countries at different times.

However, because `id` is in fact nested inside `country`, in this case, estimating the random effects as cross classified will be more time consuming, but will give us equivalent results to a three level model.

3.3.1 Standard (Less Computationally Efficient) Syntax

The below syntax will take a very long time to run with the full sample, and thus we have commented it out.

```
* mixed outcome t warmth physical_punishment || _all: R.country || _all: R.id

* est store crossed1
```

The documentation notes that we can use a *much* more computationally efficient version of the above command, which is what we do in these notes. The user can verify that both versions of the command will produce equivalent results.

In fact, at the end of handout we verify the similarity of both sets of syntax using a random sample.

3.3.2 Cross Classified With Computationally Efficient Syntax

```
mixed outcome t warmth physical_punishment || _all: R.country || id:

est store crossed2 // store crossed effects result
```

```
Performing EM optimization ...

Performing gradient-based optimization:
Iteration 0:   Log likelihood = -28516.314
Iteration 1:   Log likelihood = -28516.277
Iteration 2:   Log likelihood = -28516.277

Computing standard errors ...

Mixed-effects ML regression                          Number of obs =    9,000

        Grouping information
        -------------------------------------------------------------
                        |     No. of       Observations per group
         Group variable |     groups    Minimum    Average    Maximum
        ----------------+--------------------------------------------
                   _all |          1      9,000    9,000.0      9,000
                     id |      3,000          3        3.0          3
        -------------------------------------------------------------

                                                     Wald chi2(3)   = 1168.69
Log likelihood = -28516.277                          Prob > chi2    =  0.0000
```

```
------------------------------------------------------------------------------------
             outcome | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
---------------------+--------------------------------------------------------------
                   t |   .9434605    .065866    14.32   0.000     .8143654    1.072556
              warmth |   .9053924   .0380439    23.80   0.000     .8308277    .9799572
 physical_punishment |  -1.014385   .0499354   -20.31   0.000    -1.112257   -.916514
               _cons |    50.8301   .4123007   123.28   0.000       50.022    51.63819
------------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
_all: Identity               |
              var(R.country) |   3.429974    .930313      2.015668    5.836634
-----------------------------+------------------------------------------------
id: Identity                 |
                  var(_cons) |   8.608872   .4757699      7.725107     9.59374
-----------------------------+------------------------------------------------
               var(Residual) |   26.02862   .4752444      25.11363    26.97695
------------------------------------------------------------------------------
LR test vs. linear model: chi2(2) = 1260.84           Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

3.4 Three Level Model

```
mixed outcome t warmth physical_punishment || country: || id:  // 3 level w/ random intercept

est store threelevel // store random intercept model
```

```
Performing EM optimization ...

Performing gradient-based optimization:
Iteration 0:  Log likelihood = -28516.314
Iteration 1:  Log likelihood = -28516.277
Iteration 2:  Log likelihood = -28516.277

Computing standard errors ...
```

```
Mixed-effects ML regression                         Number of obs =   9,000

        Grouping information
        -------------------------------------------------------------
                        |     No. of       Observations per group
         Group variable |     groups    Minimum    Average    Maximum
        ----------------+--------------------------------------------
                country |         30        300      300.0        300
                     id |      3,000          3        3.0          3
        -------------------------------------------------------------

                                                    Wald chi2(3)  = 1168.69
Log likelihood = -28516.277                         Prob > chi2   =  0.0000

-------------------------------------------------------------------------------------
            outcome | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
--------------------+----------------------------------------------------------------
                  t |   .9434605    .065866    14.32   0.000     .8143654    1.072556
             warmth |   .9053924   .0380439    23.80   0.000     .8308277    .9799572
physical_punishment |  -1.014385   .0499354   -20.31   0.000    -1.112257    -.916514
              _cons |    50.8301   .4123007   123.28   0.000       50.022    51.63819
-------------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
country: Identity             |
                  var(_cons) |   3.429974    .930313      2.015668    5.836634
-----------------------------+------------------------------------------------
id: Identity                 |
                  var(_cons) |   8.608872   .4757699      7.725107     9.59374
-----------------------------+------------------------------------------------
               var(Residual) |   26.02862   .4752444      25.11363    26.97695
------------------------------------------------------------------------------
LR test vs. linear model: chi2(2) = 1260.84               Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

3.5 Nice Table of Results of Three Level and Cross Classified Model

```
etable, estimates(threelevel crossed2), ///
showstars showstarsnote /// show stars and note
column(estimate) // column is modelname
```

```
invalid 'showstars'
r(198);

r(198);
```

3.6 Verification of Syntax Equivalence for Cross Classified Model

```
keep if family <= 5 // random sample of families

quietly mixed outcome t warmth physical_punishment || _all: R.country || _all: R.id

est store crossed1A // less efficient syntax

quietly mixed outcome t warmth physical_punishment || _all: R.country || id:

est store crossed2A // more efficient syntax

etable, estimates(crossed1A crossed2A) ///
showstars showstarsnote /// show stars and note
column(estimate) // column is modelname
```

```
(8,550 observations deleted)




-------------------------------------------------------
                                  crossed1A   crossed2A
```

time	0.745 **	0.745 **
	(0.281)	(0.281)
parental warmth in past week	0.871 **	0.871 **
	(0.160)	(0.160)
physical punishment in past week	-1.262 **	-1.262 **
	(0.206)	(0.206)
Intercept	51.755 **	51.755 **
	(1.009)	(1.009)
var(R_country)	2.245	2.245
	(1.319)	(1.319)
var(R_id)	5.425	
	(1.843)	
var(e)	23.638	23.638
	(1.933)	(1.933)
var(_cons)		5.425
		(1.843)
Number of observations	450	450

** p<.01, * p<.05

3.7 QUESTIONS???