Multiple Methods of Longitudinal Data Analysis

DRAFT Notes

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1 Some Data

```
clear all

set seed 3846 //  set random seed

quietly set obs 10 // 10 observations

generate id = _n // id number

quietly expand 3 // expand by 3

sort id // sort by id

bysort id: generate t = _n // time variable

generate x = rnormal(10, 3) // random normal variable

generate w = rbinomial(1, .3) // random binomial variable

generate e = rnormal(0, 1) // random error

generate y = x + w + e // regression equation

drop e // drop error

list // list out the data

save longitudinal.dta, replace
```

```
     | id   t          x   w          y |
     |----------------------------------|
  1. |  1   1   13.26895   0   11.69778 |
  2. |  1   2   5.669146   1     6.4028 |
  3. |  1   3   11.32535   0   11.00579 |
  4. |  2   1   7.237092   0   6.865333 |
  5. |  2   2   12.60327   1   15.93668 |
     |----------------------------------|
  6. |  2   3   14.30695   1   13.92043 |
  7. |  3   1   6.360627   0   7.093182 |
  8. |  3   2   7.607124   0   7.378952 |
```

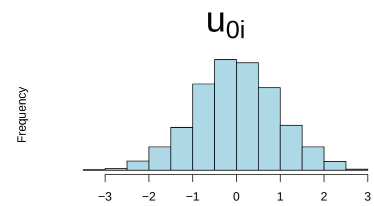
```
  9. |  3   3   11.15448   0   11.90395 |
 10. |  4   1   7.403773   1   10.07775 |
     |----------------------------------|
 11. |  4   2    11.1741   0   10.86197 |
 12. |  4   3   7.016891   0    5.84125 |
 13. |  5   1   7.085833   0   7.996722 |
 14. |  5   2   8.618052   0   9.414988 |
 15. |  5   3   10.27657   0   10.59132 |
     |----------------------------------|
 16. |  6   1   7.937543   1   10.02182 |
 17. |  6   2   12.00493   0   10.40057 |
 18. |  6   3   11.22594   1   12.66391 |
 19. |  7   1   11.34407   0   10.74489 |
 20. |  7   2   11.35657   0    11.4781 |
     |----------------------------------|
 21. |  7   3    14.3872   0   15.16246 |
 22. |  8   1   11.72829   1   11.94959 |
 23. |  8   2   8.028893   1   8.781265 |
 24. |  8   3   11.90905   1   12.49115 |
 25. |  9   1   9.205235   0   8.002105 |
     |----------------------------------|
 26. |  9   2   5.909642   1     8.8732 |
 27. |  9   3   16.80353   0   16.67801 |
 28. | 10   1   6.183664   0   6.201822 |
 29. | 10   2   7.644044   0    5.58361 |
 30. | 10   3   11.53438   0   11.32048 |
     +----------------------------------+

file longitudinal.dta saved
```

2 Multilevel Modeling

2.1 Equation

$y_{it} = \beta_0 + \beta_1 x_{it} + \beta_2 w_{it} + u_{0i} + e_{it}$

We assume that u_{0i} has a normal distribution, but do not directly estimate the values of u_{0i} for each individual.



2.2 Syntax And Results

```
use longitudinal.dta, clear

mixed y x i.w || id:
```

```
Performing EM optimization Performing gradient-based optimization:
Iteration 0:  Log likelihood = -41.789697
Iteration 1:  Log likelihood = -41.654948
Iteration 2:  Log likelihood = -41.653312
Iteration 3:  Log likelihood =  -41.65331

Computing standard errors ...

Mixed-effects ML regression                     Number of obs     =     30
Group variable: id                              Number of groups =     10
                                                Obs per group:
                                                             min =      3
                                                             avg =    3.0
                                                             max =      3
                                                Wald chi2(2)      = 236.62
Log likelihood =  -41.65331                     Prob > chi2       = 0.0000

------------------------------------------------------------------------------
           y | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
           x |    .938261   .0626023    14.99   0.000     .8155627    1.060959
         1.w |   1.682743   .3765298     4.47   0.000     .9447577    2.420728
       _cons |   .3540235   .6672312     0.53   0.596    -.9537257    1.661773
------------------------------------------------------------------------------


------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
id: Identity                 |
                  var(_cons) |   1.66e-15   1.53e-11             0           .
-----------------------------+------------------------------------------------
```

```
             var(Residual) |   .9408329   .2429279      .5671891     1.56062
------------------------------------------------------------------------------
LR test vs. linear model: chibar2(01) = 1.4e-14       Prob >= chibar2 = 1.0000
```

3 Fixed Effects

3.1 Equation

$y_{it} = \beta_0 + \beta_1 x_{it} + \beta_2 w_{it} + u_{0i} + e_{it}$

We assume that the u_{0i} are in fact, estimable. However, we end up estimating $y_{it} - \bar{y}_i = \beta_1(x_{it} - \bar{x}_i) + \beta_2(w_{it} - \bar{w}_i) + (e_{it} - \bar{e}_i)$. The u_{0i} have dropped out of this equation.

3.2 Syntax And Results

```
use longitudinal.dta, clear

xtreg y x i.w, i(id) fe
```

```
Fixed-effects (within) regression               Number of obs     =         30
Group variable: id                              Number of groups  =         10

R-squared:                                      Obs per group:
     Within  = 0.9142                                         min =          3
     Between = 0.8102                                         avg =        3.0
     Overall = 0.8673                                         max =          3

                                                F(2, 18)          =      95.93
corr(u_i, Xb) = -0.3779                         Prob > F          =     0.0000

------------------------------------------------------------------------------
           y | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
           x |    .987199   .0714222    13.82   0.000     .8371465    1.137252
         1.w |   2.757344   .5380926     5.12   0.000     1.626853    3.887834
       _cons |  -.4908022   .8026548    -0.61   0.549    -2.177117    1.195513
-------------+----------------------------------------------------------------
     sigma_u |  .87126686
     sigma_e |  .93451278
```

```
         rho |  .46501875   (fraction of variance due to u_i)
------------------------------------------------------------------------------
F test that all u_i=0: F(9, 18) = 1.59                      Prob > F = 0.1919
```

4 Difference in Differences

???

5 Cross Lagged Regression

5.1 Equation

Similar to before, there is an equation predicting y.

$y_{2i} = \beta_0 + \beta_1 y_{1i} + \beta_2 x_{1i} + \beta_3 w_{2i} + e_i$

However, we need an equation for each time point, so:

$y_{3i} = \beta_0 + \beta_1 y_{2i} + \beta_2 x_{2i} + \beta_3 w_{2i} + e_i$

And, there are also equations predicting x.

$x_{2i} = \beta_0 + \beta_1 x_{1i} + \beta_2 y_{1i} + \beta_3 w_{1i} + e_i$

$x_{3i} = \beta_0 + \beta_1 x_{2i} + \beta_2 y_{2i} + \beta_3 w_{2i} + e_i$

In cross-lagged regression, we need the data to be in wide format rather than long format.

5.2 Data Wrangling

```
use longitudinal.dta, clear

reshape wide y x w, i(id) j(t) // reshape data to wide

save longitudinalWIDE.dta, replace
```

```
(j = 1 2 3)

Data                               Long   ->   Wide
-----------------------------------------------------------------------------
Number of observations               30   ->   10
Number of variables                   5   ->   10
j variable (3 values)                 t   ->   (dropped)
xij variables:
                                      y   ->   y1 y2 y3
                                      x   ->   x1 x2 x3
                                      w   ->   w1 w2 w3
-----------------------------------------------------------------------------

file longitudinalWIDE.dta saved
```

5.3 Syntax And Results

```
use longitudinalWIDE.dta, clear

sem (y2 <- y1 x1 w1) ///
(x2 <- x1 y1 w1) ///
(y3 <- y2 x2 w2) ///
(x3 <- x2 y2 w2)
```

```
Endogenous variables
  Observed: y2 x2 y3 x3

Exogenous variables
  Observed: y1 x1 w1 w2

Fitting target model:
Iteration 0:  Log likelihood = -126.89265
Iteration 1:  Log likelihood = -126.89265

Structural equation model                                    Number of obs = 10
Estimation method: ml

Log likelihood = -126.89265
```

	Coefficient	OIM std. err.	z	P>|z|	[95% conf.	interval]
Structural						
y2						
y1	-.06638	1.722865	-0.04	0.969	-3.443134	3.310374
x1	-.1142004	1.22043	-0.09	0.925	-2.506199	2.277798
w1	.9129669	4.046881	0.23	0.822	-7.018773	8.844707
_cons	10.84123	5.48777	1.98	0.048	.0853985	21.59706
x2						
y1	1.077149	1.270605	0.85	0.397	-1.413191	3.567489
x1	-1.050991	.9000609	-1.17	0.243	-2.815078	.7130958
w1	-.2019809	2.984554	-0.07	0.946	-6.0516	5.647638
_cons	8.580689	4.047204	2.12	0.034	.6483162	16.51306
y3						
y2	.6385229	.7520661	0.85	0.396	-.8354996	2.112545
x2	-.6869322	.9228791	-0.74	0.457	-2.495742	1.121878
w2	.6030497	2.657096	0.23	0.820	-4.604762	5.810862
_cons	12.06822	3.779402	3.19	0.001	4.660724	19.47571
x3						
y2	.7641998	.6168695	1.24	0.215	-.4448422	1.973242
x2	-.8415779	.756976	-1.11	0.266	-2.325223	.6420677
w2	.6179903	2.179438	0.28	0.777	-3.65363	4.88961
_cons	12.10439	3.099991	3.90	0.000	6.028522	18.18026
var(e.y2)	7.584673	3.391969			3.156952	18.22241
var(e.x2)	4.125296	1.844888			1.717063	9.911148
var(e.y3)	6.049814	2.705559			2.518101	14.53486
var(e.x3)	4.070208	1.820252			1.694134	9.778798

LR test of model vs. saturated: chi2(10) = 85.21 Prob > chi2 = 0.0000

6 Summary [1]

```
Method <- c("Multilevel Modeling",
            "Fixed Effects",
            "Cross Lagged Regression")

`Control for Time Invariant Observed` <- c("yes",
                                            "yes",
                                            "yes")

`Control for Time Varying Observed` <- c("yes",
                                          "yes",
                                          "yes")

`Control for Time Invariant Unobserved` <- c("partially",
                                              "yes",
                                              "no")

`Control for Time Varying Unobserved` <- c("no",
                                            "no",
                                            "no")
`Estimate Reciprocal Causality` <- c("no",
                                      "no",
                                      "yes")

`Control for Earlier or Baseline y` <- c("automatic",
                                          "automatic",
                                          "must explicitly specify")

mytable <- data.frame(Method,
                      `Control for Time Invariant Observed`,
                      `Control for Time Varying Observed`,
                      `Control for Time Invariant Unobserved`,
                      `Control for Time Varying Unobserved`,
                      `Estimate Reciprocal Causality`,
                      `Control for Earlier or Baseline y`,
                      check.names = FALSE)

pander:: pander(mytable)
```

Table 1: Table continues below

Method	Control for Time Invariant Observed
Multilevel Modeling	yes
Fixed Effects	yes
Cross Lagged Regression	yes

Table 2: Table continues below

Control for Time Varying Observed	Control for Time Invariant Unobserved
yes	partially
yes	yes
yes	no

[1] Some of the decisions in this table are arguable.

Table 3: Table continues below

Control for Time Varying Unobserved	Estimate Reciprocal Causality
no	no
no	no
no	yes

Control for Earlier or Baseline y
automatic
automatic
must explicitly specify