The Logistic Curve Is A Theoretical Construct

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1 The Logistic Curve

> 💡 Equations
>
> The equation for the curve below comes from the logistic regression equation:
>
> $$\ln\left(\frac{p(\text{outcome})}{1 - p(\text{outcome})}\right) = \beta_0 + \beta_1 x_1$$
>
> We exponentiate both sides of the equation:
>
> $$\frac{p(\text{outcome})}{1 - p(\text{outcome})} = e^{\beta_0 + \beta_1 x_1}$$
>
> We multiply both sides by the denominator of the fraction that is on the left hand side of the equation:
>
> $$p(\text{outcome}) = e^{\beta_0 + \beta_1 x_1}(1 - p(\text{outcome}))$$
>
> Then:
>
> $$p(\text{outcome}) = e^{\beta_0 + \beta_1 x_1} - e^{\beta_0 + \beta_1 x_1} * p(\text{outcome})$$
>
> Then:
>
> $$p(\text{outcome}) + e^{\beta_0 + \beta_1 x_1} * p(\text{outcome}) = e^{\beta_0 + \beta_1 x_1}$$
>
> Then:
>
> $$(1 + e^{\beta_0 + \beta_1 x_1}) * p(\text{outcome}) = e^{\beta_0 + \beta_1 x_1}$$

And, finally:

$$p(\text{outcome}) = \frac{e^{\beta_0 + \beta_1 x_1}}{1 + e^{\beta_0 + \beta_1 x_1}}$$

```
twoway ///
(function y = exp(x)/(1 + exp(x)), range(-10 10) lwidth(thick) lpattern(dash)), ///
title("The Logistic Curve") ///
ytitle("Probablity") ///
scheme(stcolor)

graph export logistic.png, width(2000) replace
```
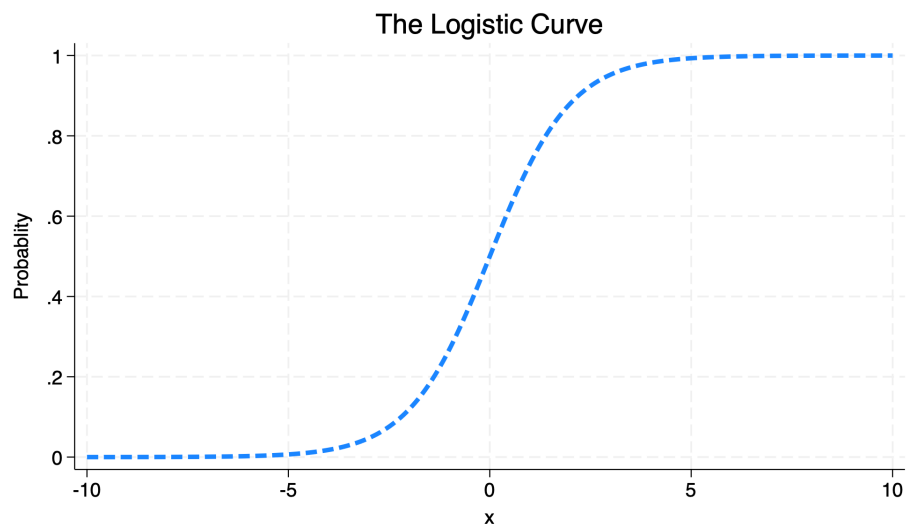

Figure 1: The Logistic Curve

2 The Logistic Curve is a *Theoretical Construct*

> 💡 Tip
>
> Any given data set usually maps to only part of the full logistic curve.

```
twoway ///
(function y = exp(x)/(1 + exp(x)), range(-10 10) lwidth(thick) lpattern(dash)) /// full logisti
(function y = exp(x)/(1 + exp(x)), range(2 5) lwidth(vvthick)) /// sample data set
(function y = exp(x)/(1 + exp(x)), range(-1 1) lwidth(vvthick)) /// sample data set
(function y = exp(x)/(1 + exp(x)), range(-3 -2) lwidth(vvthick) lcolor(orange)), /// sample dat
```

```
title("Any Given Data Set") ///
ytitle("Probablity") ///
subtitle("Usually Maps To Only Part Of The Full Logistic Curve") ///
legend(order(1 "full logistic curve" 2 "sample data set" 3 "sample data set" 4 "sample data set
scheme(stcolor)

graph export logistic2.png, width(2000) replace
```
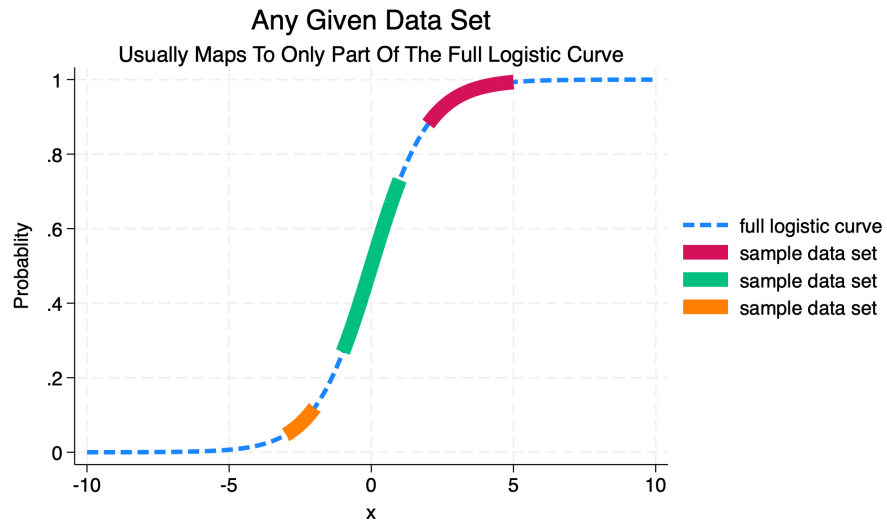

Figure 2: The Logistic Curve Is A Theoretical Construct