More On The Cox Proportional Hazards Model

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DRAFT VERSION: COMMENTS, QUESTIONS AND CORRECTIONS WELCOME.

Background

This handout draws on ideas that appear in many textbooks and sets of notes on survival analysis. I have seen them most clearly presented in notes (nd) by Professor Ronghui Xu and *An Introduction to Event History Analysis* (2007) from The Oxford Summer School. However, I present these ideas using the notation preferred by Paul Allison, and by the Stata documentation, which I think is more clear.

The Hazard

Per Paul Allison:

For a single covariate.

$$\ln(h(t)) = a(t) + \beta_1 x_1$$

Some Algebra (Per Oxford Summer School)

Then

$$h(t) = e^{a(t)+\beta_1 x_1}$$

Then

$$h(t) = e^{a(t)} e^{\beta_1 x_1}$$

Per ideas presented by Professor Xu, and elsewhere, we note that "the baseline hazard depends on t but not on the covariates," and, as we will see below, "the hazard ratio, depends on the covariates, but not on t.

The Hazard Ratio (Per Allison and The Oxford Summer School)

We use a dichotomous covariate for the sake of illustration.

$$\text{HR} = \frac{h(t)|x = 1}{h(t)|x = 0} =$$

$$\frac{e^{a(t)}e^{\beta_1 \times 1}}{e^{a(t)}e^{\beta_1 \times 0}} =$$

$$\frac{e^{\beta_1}}{e^0} = e^{\beta_1}$$